

# Tunable Fermi-Edge Resonance in an Open Quantum Dot

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Resonant tunneling in an open mesoscopic quantum dot is proposed as a vehicle to realize a tunable Fermi-edge resonance with variable coupling strength. We solve the x-ray edge problem for a generic nonseparable scatterer and apply it to describe tunneling in a quantum dot. The tunneling current power law exponent is linked to the S-matrix of the dot. The control of scattering by varying the dot shape and coupling to the leads allows to explore a wide range of exponents. Transport properties, such as weak localization, mesoscopic conductance fluctuations, and sensitivity to Wigner-Dyson ensemble type, have their replicas in the Fermi-edge singularity.

Quantum dots host a number of interesting quantum transport phenomena, such as Coulomb blockade [1,2], Kondo effect [3] weak localization and universal conductance fluctuations [4]. Electron scattering inside the dot as well as the dot-lead coupling can be controlled externally by gates, which makes it possible to reveal phenomena of interest by varying system parameters. Such tunability has been exploited to demonstrate [5–8] new exotic varieties of Kondo effect. In this article we propose to employ mesoscopic dots, in a similar controllable fashion, to study the Fermi-edge singularity (FES).

FES is a fundamental manifestation of many-body physics taking place when an electron with energy just above the Fermi level tunnels into a metal, while leaving a localized hole behind. After tunneling, the electron forms a quasiresonance due to interaction with the hole. This strongly affects the transition rate which is typically found to be a power law function of Mahan-Nozieres-deDominicis form  $A(\epsilon) \propto (\epsilon - \epsilon_F)^{-\alpha}$ . Similar to the Kondo problem, the FES problem [9,10] is one of few many-body problems exactly solvable in the nonperturbative regime of strong interaction.

First discovered in the 60's in the context of x-ray absorption in metals [9,10], the FES physics has found many other applications. In 1992, Matveev and Larkin [11] considered resonant tunneling and predicted a power law singularity, identical to FES, as a function of the resonance position relative to the Fermi level. In this case, the exponent  $\alpha$  in the tunneling  $I - V$  characteristic is controlled by interaction of the tunneling electron and localized hole. The latter is system-specific, and depends on scattering phases and screening via Friedel sum rule.

Below we generalize the theory [11] to describe resonant tunneling into an open quantum dot. Chaotic scattering in the dot, returns the tunneling electron many times to the hole, which enhances the FES singularity and makes it ‘tunable’, i.e. scattering-dependent. (In a noninteracting mesoscopic system [12], multiple returns to a resonant level are known to produce weak localization and conductance fluctuations.) While charging effects may interfere with resonant tunneling [13,14], in open dots one can ignore charge fluctuations and focus on the interplay of scattering and interaction with localized

hole which forms FES.

Manifestations of FES have been observed in tunnel junctions [15] and in low temperature telegraph noise [16]. The role of scattering in quantum dots can be studied by resonant tunneling spectroscopy [17].

The canonical theory [10], based on separable scatterer model, is difficult to adapt to mesoscopic scattering. The crucial problem arises from noncommutativity of the S-matrices before and after electron release in the dot, rendering the separable model, along with the bosonization approach [18] used to handle it, irrelevant. Our approach builds on the Yamada and Yosida theory [19] of Anderson orthogonality for multichannel nonseparable scatterer, recently advanced by Muzykantskii *et al.* [20,21], as well as on Matveev phase shift approach [22] to charge fluctuations in quantum dots.

The theory presented below yields an exact relation of the FES exponent with the quantum dot S-matrix and reveals that the exponent structure is similar to that in the separable scatterer case. The orthogonality catastrophe due to Fermi sea shakeup by switching of charge state at tunneling accounts only for one, negative part of the FES exponent, while the leading, positive part arises from interaction in the final state. Applying the result to the quantum dot problem, we find that by varying the dot scattering parameters the exponent  $\alpha$  can be tuned to any value in the weak or strong coupling regime.

Our results for open dots complement the work on the orthogonality catastrophe [23–25] and FES [26] in closed quantum dots which use the exact one particle states and energies to express the many-body overlap and transition rate. The enhancement of orthogonality by disorder, discussed by Gefen *et al.* [25], has the same underlying physics as our backscattering-enhanced FES.

The geometry of interest is pictured in Fig. 1 (a). We consider tunneling from a small dot which holds few electrons and has a large charging energy, into an open mesoscopic dot. The latter is characterized by a  $N \times N$  S-matrix, where  $N$  is the number of channels connecting the dot and the leads. The interaction of electrons in the open dot with the localized hole in the small dot is described [22] by the backscattering phase  $\delta$  in the channel connecting the two dots (Fig. 1 (b)).

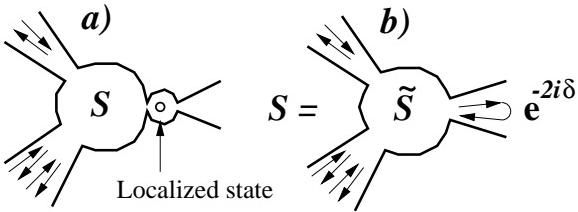


FIG. 1. a) An open quantum dot weakly coupled to a small closed dot which holds a localized electron that can tunnel into the open dot. b) Relation of the open dot S-matrix  $S$  and an auxiliary matrix  $\tilde{S}$  is illustrated. The latter describes the dot with an extra open channel added to incorporate backscattering on the small dot charge state.

How does the dot S-matrix depend on backscattering? The answer is found most easily by considering an auxiliary scattering problem with an additional channel which describes the point contact between the dots. This defines an extended S-matrix  $\tilde{S}$  of size  $(N+1) \times (N+1)$ . The physical matrix  $S$  can be linked with the auxiliary matrix  $\tilde{S}$  by imposing the quasiperiodicity relation  $u_{N+1}^{(out)} = e^{-2i\delta} u_{N+1}^{(in)}$  on the in and out components of the added channel, and eliminating these components from the scattering relation  $u_i^{(out)} = \tilde{S}_{ij} u_j^{(in)}$ . We obtain

$$S_{ij} = \tilde{S}_{ij} + \frac{\tilde{S}_{i(N+1)} \tilde{S}_{(N+1)j}}{e^{-2i\delta} - r}, \quad i, j = 1 \dots N \quad (1)$$

with  $r \equiv \tilde{S}_{(N+1)(N+1)}$  the backscattering amplitude in the extended picture. One can verify that  $S$ , defined by (1), is unitary provided that  $\tilde{S}$  is unitary. The relation between  $S$  and  $\tilde{S}$  is illustrated graphically in Fig. 1 (b).

We emphasize that the parameters  $r$  and  $\delta$  which appear together in Eq.(1) and below describe different physics and, in particular, arise on different length scales. The phase shift  $\delta$  is a constant determined by the effects within a screening length from localized hole. In contrast, the quantity  $r$ , describing transport in the interior of the dot, is sensitive to the dot shape, and thus is tunable.

The utility of the  $\delta$ -dependent S-matrix (1) can be assessed by using the result for the orthogonality catastrophe with nonseparable scatterer. In the latter problem, one is interested in the overlap of the many-body ground states with different  $S = S_{0,1}$ . Yamada and Yosida [19] derived a remarkably simple formula for this overlap,

$$\langle 1|0 \rangle = \exp \left( -\frac{\beta^2}{2} \ln N \right), \quad \beta^2 = \frac{1}{4\pi^2} |\text{tr} \ln^2 S_1 S_0^{-1}| \quad (2)$$

with  $N$  the number of particles per scattering channel.

For two different phases  $\delta$ ,  $\delta'$  in (1), the compound matrix  $R = S(\delta') S^{-1}(\delta)$  that appears in (2) is equal to

$$R_{ij} = \delta_{ij} + \left( \frac{U(\delta)}{U(\delta')} - 1 \right) v_i^* v_j, \quad U(\delta) = \frac{e^{-2i\delta} - r}{e^{-2i\delta} r^* - 1} \quad (3)$$

with  $v_i = \tilde{S}_{(N+1)i} (1 - |r|^2)^{-1/2}$ ,  $i = 1 \dots N$ , the normalized column of  $\tilde{S}$ . Remarkably,  $R$  differs from a unit matrix

only by a matrix of rank one. It means that, despite the complex dependence (1) on the scattering phase  $\delta$  that appears to affect the entire  $N \times N$  matrix  $S$ , the orthogonality problem is effectively a single channel-like. The overlap  $\langle 0|1 \rangle$  is described by Eq.(2) with

$$\beta = \frac{1}{2\pi} \text{Im} \ln \left( \frac{U(\delta)}{U(\delta')} \right) \quad (4)$$

Thus  $\beta$  depends on transport in the dot solely via the backscattering amplitude  $r$ . Both the modulus of  $r$  and its phase, being functions of the dot shape and dot-lead transmissions, are under experimental control, and thus  $\beta$  can be tuned to any value in the interval  $0 < \beta < 1$ .

We analyze the FES problem below for the scatterers (1), and find a relation between the FES and the orthogonality exponents identical to the single channel case,

$$\alpha = 2\beta - \beta^2 \quad (5)$$

This is not entirely unexpected, given that the above analysis reveals hidden single-channel character of the orthogonality problem. However, since the canonical FES theory [9,10] is limited to the separable scatterer situation, the relation (5) cannot be deduced directly. Instead, we shall develop an approach for a generic S-matrix, and then specialize to the quantum dot case (1).

Turning to the analysis of the FES problem, we consider the tunneling electron Green's function [9,10]

$$G(\tau) = \text{tr} \left( \hat{\psi}(0) e^{-i\mathcal{H}_1 \tau} \hat{\psi}^+(0) e^{i\mathcal{H}_0 \tau} \hat{\rho}_e \right) \quad (6)$$

with interaction included in the Hamiltonians  $\mathcal{H}_{1,2}$  which describe electron scattering by the charged/uncharged localized state. Here  $\hat{\psi}(\tau) = \sum_\alpha u_\alpha \hat{a}_\alpha(\tau)$  is the operator of a tunneling electron with  $\hat{a}_\alpha$  labelled by energy and scattering channel, while  $\hat{\rho}$  is electron density matrix

$$\hat{\rho}_e = \frac{1}{Z} \exp \left( -\beta \sum_\alpha \epsilon_\alpha \hat{a}_\alpha^\dagger \hat{a}_\alpha \right), \quad \beta^{-1} = k_B T \quad (7)$$

with the normalization factor  $Z = \prod_\alpha (1 + e^{-\beta \epsilon_\alpha})$ . The original approach [10] employs a diagrammatic expansion of (6) and, using the closed loop calculus, expresses it through electron Green's function describing time-dependent scattering at  $0 < t < \tau$ . Then the series for the Green's function are resummed in order to replace the scattering potential by the S-matrix. The resummed series, treated using Dyson equation in the time domain, lead to a singular integral equation that can be solved using a particular variety of the Wiener-Hopf method.

Here we proceed differently, trying to avoid the diagrammatic expansion altogether. This has a two-fold advantage. Firstly, we shall be able to introduce the single particle S-matrices at an early stage of the calculation, thereby bypassing the resummation problem. Secondly, our approach will apply to noncommuting S-matrices.

As a first step, we use the commutation relations

$$\hat{a}_\alpha^+ \hat{\rho}_e = e^{\beta\epsilon_\alpha} \hat{\rho}_e \hat{a}_\alpha^+, \quad \hat{a}_\alpha^+ e^{i\mathcal{H}_0\tau} = e^{-i\epsilon_\alpha\tau} e^{i\mathcal{H}_0\tau} \hat{a}_\alpha^+ \quad (8)$$

to rewrite Eq. (6) as

$$G(\tau) = \sum_{\alpha, \alpha'} u_{\alpha'}^* u_\alpha e^{\beta\epsilon_{\alpha'}} e^{-i\epsilon_{\alpha'}\tau} \text{tr} (e^{-i\mathcal{H}_1\tau} e^{i\mathcal{H}_0\tau} \hat{\rho}_e \hat{a}_{\alpha'}^+ \hat{a}_\alpha) \quad (9)$$

where  $\alpha, \alpha'$  label single particle band states.

Next, we note that the quantities  $e^{-i\mathcal{H}_1\tau}$ ,  $e^{i\mathcal{H}_0\tau}$ ,  $\hat{\rho}_e$  are exponentials of operators quadratic in  $\hat{a}_\alpha$ ,  $\hat{a}_{\alpha'}^+$ , which allows to write their product as

$$e^{-i\mathcal{H}_1\tau} e^{i\mathcal{H}_0\tau} \hat{\rho}_e = Z^{-1} \exp \left( \sum_{\beta, \beta'} w_{\beta\beta'} \hat{a}_\beta^+ \hat{a}_{\beta'} \right) \quad (10)$$

where the operator  $\hat{w}$ , defined by Eq. (10) and to be found in an explicit form below, acts in the single electron Hilbert space. With the help of the definition (10) the trace in Eq. (9) can be expressed through  $\hat{w}$  as follows:

$$\text{tr} (e^{-i\mathcal{H}_1\tau} e^{i\mathcal{H}_0\tau} \hat{\rho}_e \hat{a}_{\alpha'}^+ \hat{a}_\alpha) = \frac{\det (\hat{1} + e^{\hat{w}})}{Z} (\hat{1} + e^{-\hat{w}})^{-1}_{\alpha\alpha'}$$

which reduces the FES problem to analyzing the operators  $\hat{1} + e^{\pm\hat{w}}$ . As we find shortly, the latter are related to the single-particle S-matrix and energy distribution. The electron statistics is thus fully accounted for by the algebra involved in the construction of the operator  $\hat{1} + e^{\hat{w}}$  and its determinant, while the solution of the time-dependent scattering problem amounts to computing the inverse  $(\hat{1} + e^{-\hat{w}})^{-1}$ . The explicit separation of the many-body and the single-particle effects provides an efficient treatment of the FES problem.

To make progress, we use the idea of Ref. [27] to link  $e^{\hat{w}}$  with single-particle quantities. From Baker-Hausdorff series for  $\ln(e^A e^B)$  in terms of multiple commutators of  $A$  and  $B$ , noting the correspondence between the commutator algebra of the many-body operators quadratic in  $a_\alpha$ ,  $a_{\alpha'}^+$  and the single-particle operators, we find

$$e^{\hat{w}} = e^{-i\hat{h}_1\tau} e^{i\hat{h}_0\tau} e^{-\beta\hat{\epsilon}} \quad (11)$$

Here the operators  $\hat{h}_{0,1}$  and  $e^{-\beta\hat{\epsilon}}$  are related to the single particle Hamiltonian and density matrix (7) as

$$\mathcal{H}_{0,1} = \sum_{\alpha\alpha'} (\hat{h}_{0,1})_{\alpha\alpha'} a_\alpha^+ a_{\alpha'}, \quad e^{-\beta\hat{\epsilon}} = e^{-\beta\epsilon_\alpha} \delta_{\alpha\alpha'} \quad (12)$$

With the help of the result (11), defining  $\hat{n} = (1 + e^{\beta\hat{\epsilon}})^{-1}$ , the determinant  $\det (1 + e^{\hat{w}})$  can be brought to the form

$$\det (1 + e^{\hat{w}}) = Z \det (1 - n(\epsilon) + e^{-i\hat{h}_1\tau} e^{i\hat{h}_0\tau} n(\epsilon)) \quad (13)$$

The operator  $e^{-i\hat{h}_1\tau} e^{i\hat{h}_0\tau}$  is represented most naturally in the basis of time-dependent scattering states constructed as wavepackets labeled by the time of arrival at the scatterer. As Fig. 2 illustrates, the result of backward-and-forward time evolution

$$\hat{S} \equiv e^{-i\hat{h}_1\tau} e^{i\hat{h}_0\tau} = \delta_{t,t'} \times \begin{cases} R, & 0 < t < \tau \\ 1, & \text{else} \end{cases} \quad (14)$$

with  $R = S_1 S_0^{-1}$  a compound S-matrix, and  $S_{0,1}$  the S-matrices for the charged/uncharged localized state. (In the single-channel case,  $R = e^{2i(\delta-\delta')}$ .) Thus we rewrite Eq.(13) as  $\det (1 + e^{\hat{w}}) = Z \det (1 + (\hat{S} - 1)\hat{n})$ . Similarly, the operator  $(1 + e^{-\hat{w}})^{-1}$  becomes

$$(1 + e^{-\hat{w}})^{-1} = (n(\epsilon) + (1 - n(\epsilon))\hat{S}^{-1})^{-1} n(\epsilon) \quad (15)$$

with  $n$ ,  $S$  being operators in the Hilbert space of functions of time.

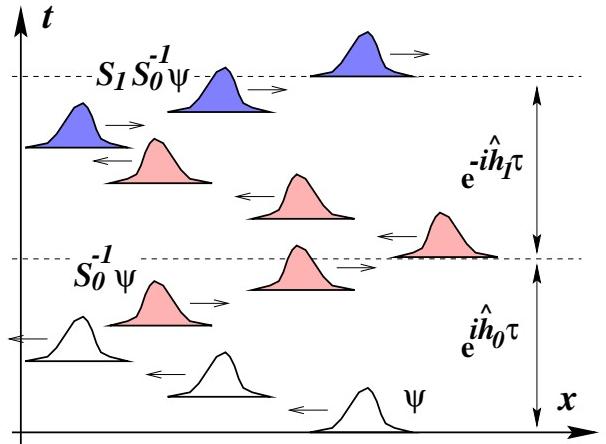


FIG. 2. Schematic forward and backward scattering time evolution  $e^{-i\hat{h}_1\tau} e^{i\hat{h}_0\tau}$ , with ballistic transport outside the scattering region. The compound S-matrix  $R = S_1 S_0^{-1}$  accounts for sequential scattering described by the S-matrices  $S_{0,1}$  corresponding to the hamiltonians  $\hat{h}_{0,1}$ .

Thus the Green's function (9) is brought to the form

$$G(\tau) = L e^C, \quad e^C = \det (1 + (\hat{S} - 1)\hat{n}) \quad (16)$$

$$L = \sum_{\epsilon, \epsilon'} e^{-i\epsilon'\tau} \langle \tilde{u}_\epsilon | (1 - \hat{n}) (\hat{n} + \hat{S}^{-1}(1 - \hat{n}))^{-1} | \tilde{u}_\epsilon^* \rangle \quad (17)$$

where  $\tilde{u}_\epsilon = \sum_\alpha u_\alpha \delta(\epsilon - \epsilon_\alpha)$  is a vector in channel space, and  $\langle \dots \rangle$  includes summation over scattering channels. (A related determinant identity for  $e^C$  has been known in the theory of counting statistics [28,20].)

The factorization  $G(\tau) = L e^C$  demonstrated for a general scattering problem with noncommuting  $S_{0,1}$ , provides connection with Nozieres-deDominicis theory and generalizes it to nonseparable scatterers. The two factors in  $G(\tau)$ , expressed through single-particle quantities, in the language of Ref. [10] correspond to the contributions of the open line and closed loop diagrams, respectively.

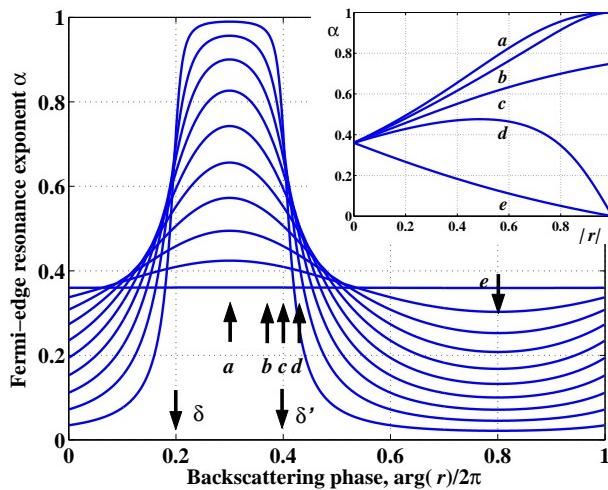


FIG. 3. The dependence (5),(4) of the tunneling exponent  $\alpha$  on  $\arg(r)$  for  $|r| = 0, 0.1, \dots, 0.9$ ; *Inset:* The exponent  $\alpha$  vs.  $|r|$  for several values  $\arg(r)$  marked by arrows (*a*–*e*).

An explicit result for  $G(\tau)$  now follows by noting that, with respect to channel indices, the operators in (16),(17) are diagonal in the eigenbasis of  $R = S_1 S_0^{-1}$ , where  $L$  is additive, while  $e^C$  is multiplicative. Using the standard singular integral equation solution [10,19], we obtain

$$L = - \sum_j |u_j|^2 \frac{i}{\tau} (-i\tau\xi_0)^{2\beta_j}, \quad e^C = e^{-i\delta\mu t} (-i\tau\xi_0)^{-\sum_j \beta_j^2}$$

at  $t < \hbar/T$ , with  $e^{2\pi i \beta_j}$  the eigenvalues of  $R$ , and  $\xi_0 \simeq E_F$  the ultraviolet cutoff. The prefactor  $e^{-i\delta\mu t}$  describing the localized state energy offset can be discarded.

In the case of our primary interest (3),  $R$  has only one nontrivial eigenvalue  $e^{2\pi i \beta}$ , given by (4), and  $u_i$  is an eigenvector  $v_i$ . This gives  $G(\tau) \propto \tau^{-(1-\beta)^2}$ , leading directly to the result (5). The dependence of the FES exponent  $\alpha$  on  $|r|$  and  $\arg r$  is displayed in Fig. 3.

The effect of mesoscopic fluctuations on FES can be described by drawing  $\tilde{S}$  from a Wigner-Dyson ensemble of matrices of size  $(N+1) \times (N+1)$ , orthogonal, unitary or symplectic, depending on the symmetry. The backscattering amplitude  $r$ , being a diagonal matrix element of  $\tilde{S}$ , has a distribution [29]  $P(r) \propto (1-|r|^2)^\gamma$  with  $\gamma = N+1, (N+2)/2, 2N+2$  for the three ensembles. This generates an FES exponent distribution of width  $\simeq \gamma^{-1/2}$  which is small at large  $N$ . For fixed modulus  $|r|$ , the change of the FES exponent can be of either sign depending on the phase  $\theta = \arg r$  (Fig. 3). The effect of scattering is particularly prominent at  $|r|$  approaching 1, where the FES is strongly enhanced for the phase values  $\theta$  between  $\delta$  and  $\delta'$ , and suppressed otherwise, which corresponds to resonance formation in the dot.

In summary, this work presents an exact solution of the Fermi-edge resonance problem for noncommuting scatterers, relevant for tunneling in mesoscopic systems. We consider an application to resonant tunneling in open quantum dots and show that a resonance with tunable

interaction strength, and thus with a variable power law exponent, can be realized. The resonance is strongly enhanced by backscattering in a phase-sensitive fashion.

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